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Effective Potential of Super Yang-Mills Theory on $M^4 \times S^1$ and related topics [1]

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Abstract

We study the gauge symmetry breaking of an $\mathcal{N}=1$ supersymmetric Yang-Mills theory defined on $M^4 \times S^1$, taking correctly account of the vacuum expectation values for the adjoint scalar field $\langle\Sigma\rangle$ in vector multiplet in addition to the Wilson line phases $\langle A_y\rangle$. We evaluate the one-loop effective potential and obtain the vacuum configuration, for which an $SU(N)$ gauge symmetry is not broken. In case of an orbifold S^1/Z_2, under appropriate orbifolding boundary conditions, two Higgs doublets are embedded in the zero modes, $A_y^{(0)}$ and $\Sigma^{(0)}$. We point out that the tree-level scalar potential resulted from the covariant derivative for the adjoint scalar field is identical to the D-term of the MSSM. We also briefly mention the mass spectra of the gauge and Higgs sector in the theory.

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1 Introduction

Gauge theory in higher dimensions is able to provide new approaches to the long standing problems in high energy physics. One of the interesting ideas in the higher dimensional gauge theory is the gauge-Higgs unification [1], where scalar fields are unified into the higher dimensional gauge field as extra components. The extra component, in fact, behaves as the scalar field at low energies.

The zero mode of the extra component of the gauge field becomes dynamical variable to induce the vacuum expectation values, reflecting the topology of the extra dimension. The vacuum expectation values are closely related to the Wilson line phase, and one usually evaluates the effective potential for the phases and the vacuum expectation values is determined dynamically. The zero mode is massless at the tree-level, but it acquires mass term at quantum level. The mass term is obtained from the effective potential.

If we can identify the Higgs scalar in the standard model as the extra component of the gauge field, namely, its zero mode, the arbitrariness of the Higgs sector in the standard model is partially solved. The Higgs self interactions appears from the higher dimensional gauge coupling, and more interestingly, the gauge hierarchy problem is resolved. This is because the Higgs mass is generated through quantum corrections to be finite from the effective potential for the Wilson line phases. The dynamics of the Wilson line phases, the Hosotani mechanism [2] plays the crucial role.

We study an $\mathcal{N} = 1$ vector multiplet on $M^4 \times S^1$ and obtain the one-loop effective potential, taking account of the vacuum expectation values for the adjoint scalar field, which has been overlooked in the past, in addition to the Wilson line phases [3]. We also study the case of two Higgs doublets by proceeding to an orbifold S^1/Z_2 and point out that the scalar potential is identical to that of the MSSM with $g_Y = \sqrt{3}g_2$. We briefly mention the mass spectra of the gauge and Higgs sector.

2 Effective potential $V_{eff}(\langle A_y \rangle, \langle \Sigma \rangle)$

Let us consider an $\mathcal{N} = 1$ vector multiplet $(A_{\hat{\mu}}, \Sigma, \lambda_D)$ on $M^4 \times S^1$. As is well known, reflecting the topology of S^1, the order parameter for the gauge symmetry breaking is given by the zero mode of the extra component of the gauge potential $A_y^{(0)}$. In addition to it, one should not overlook the adjoint scalar field Σ, which also carries the colour indices. Hence, there are two kinds of the order parameters for the gauge symmetry breaking,

$$\langle A_y^{(0)} \rangle = \text{Wilson line phases} \qquad \text{and} \qquad \langle \Sigma \rangle \in \text{Adj. representation}. \tag{1}$$

In order to study the vacuum structure of the theory, one needs to take both into account. We expand fields around the VEV's,

$$A_{\hat{\mu}} = \langle A_{\hat{\mu}} \rangle \delta_{\hat{\mu} y} + \bar{A}_{\hat{\mu}}, \qquad \Sigma = \langle \Sigma \rangle + \bar{\Sigma}. \tag{2}$$

Thanks to $\langle\Sigma\rangle$, the tree-level potential arises from the covariant derivative for Σ,

$$V_{tree} = -g^2 \mathrm{tr}[\langle A_y\rangle,\ \langle\Sigma\rangle]^2. \tag{3}$$

By utilizing the $SU(N)$ degrees of freedom, we can diagonalize $\langle A_y\rangle$ as

$$gL\langle A_y\rangle = (\theta_1, \theta_2, \cdots, \theta_N) \quad \text{with} \quad \sum_{i=1}^{N} \theta_i = 0. \tag{4}$$

It is natural to expect that the vacuum configuration satisfies the flatness condition, $[\langle A_y\rangle,\ \langle\Sigma\rangle] = 0$, so that we can parametrize $\langle\Sigma\rangle$ as

$$\langle\Sigma\rangle = \mathrm{diag.}(\sigma_1, \sigma_2, \cdots, \sigma_N) \quad \text{with} \quad \sum_{i=1}^{N} \sigma_i = 0. \tag{5}$$

The contribution to the VEV's from bosons and fermions cancels due to the supersymmetry, so that the effective potential vanishes. One needs to break the supersymmetry in order to obtain the nonvanishing effective potential. There is a simple framework to break supersymmetry in studying the dynamics of the Wilson line phases. That is the Scherk-Schwarz (SS) mechanism [4], by which the boundary condition of λ_D in the vector multiplet for the S^1 direction is twisted by an amount of β,

$$\lambda_D(y+L) = \mathrm{e}^{2\pi\beta}\lambda_D(y). \tag{6}$$

The other fields satisfy the periodic boundary condition. The nontrivial values for β explicitly breaks supersymmetry.

One can also introduce the gauge invariant mass M for λ_D to break supersymmetry. In this case, however, one should notice that the σ_i-dependent divergent terms like $M\langle\Sigma\rangle^2\Lambda$ appear to spoil the desirable nature of the ultraviolet insensitivity for the effective potential. One can formally remove the divergent by subtracting the $n=0$ mode in the KK mode summation, which corresponds to the contribution from $L\to\infty$. Here we consider the SS mechanism alone, so that the effective potential is independent of the ultraviolet cutoff.

By the straightforward calculations [5], we arrive at

$$\begin{aligned} V_{eff}(\sigma,\theta) &= \frac{-4\times 2}{(2\pi)^{\frac{5}{2}}} \sum_{i,j=1}^{N}\sum_{n=1}^{\infty} \left(\frac{g^2\sigma_{ij}^2}{n^2L^2}\right)^{\frac{5}{4}} K_{\frac{5}{2}}\left(\sqrt{(g\sigma_{ij}nL)^2}\right)[1-\cos(2\pi n\beta)] \\ &\times\ 2\cos[n(\theta_i-\theta_j)], \qquad \sigma_{ij} \equiv \sigma_i - \sigma_j, \end{aligned} \tag{7}$$

where the modified Bessel function is expressed as

$$K_{\frac{5}{2}}(y) = \left(\frac{\pi}{2y}\right)^{\frac{1}{2}}\left(1+\frac{3}{y}+\frac{3}{y^2}\right)\mathrm{e}^{-y}. \tag{8}$$

The Boltzmann like suppression factor $\mathrm{e}^{-gn\sigma_{ij}L}$ is understood from the similarity of the effective potential as that at finite temperature. Particles with smaller wavelengths

than the inverse temperature ($\sim L$) have the Boltzmann suppressed distribution in the system. It has been known that the factor is important for gauge symmetry breaking through the Wilson line phases [6][7].

Noting that

$$0 \leq K_{\frac{5}{2}}\left(\sqrt{(g\sigma_{ij}nL)^2}\right)[1-\cos(2\pi n\beta)] \quad \forall \sigma_{ij}, \beta, \tag{9}$$

we immediately see that the vacuum configuratio is given by

$$(\theta_i, \sigma_i) = \left(\frac{2\pi k}{N}, 0\right) \quad k = 0, 1, \cdots, N-1. \tag{10}$$

Since the configuration for θ_i is the center of $SU(N)$, the $SU(N)$ gauge symmetry is unbroken for the vacuum configuration.

The zero modes $A_y^{(0)a=3}$ and $\Sigma^{a=3}$ become massive at one-loop level, and their masses, for example $SU(2)$ case, are obtained by the mass matrix,

$$\mathcal{M}^2_{A_y,\Sigma} = \left(\begin{array}{cc} \frac{\partial^2 V_{eff}}{\partial\theta^2} & \frac{\partial^2 V_{eff}}{\partial\theta\partial\sigma} \\ \frac{\partial^2 V_{eff}}{\partial\theta\partial\sigma} & \frac{\partial^2 V_{eff}}{\partial\sigma^2} \end{array}\right)_{\theta=0,\sigma=0}. \tag{11}$$

The off-diagonal elements vanishes for the vacuum configuration, and we obtain that

$$m^2_{A_y^{(0)a=3}} = 3m^2_{\Sigma^{a=3}} = \left(\frac{g_2}{L}\right)^2 \frac{6}{\pi^2} g(\beta), \quad g(\beta) = \sum_{n=1}^{\infty} \frac{1}{n^3}\left[1-\cos(2\pi n\beta)\right], \tag{12}$$

where $g_2 \equiv g/\sqrt{2\pi R}$.

3 An orbifold S^1/Z_2 case

Let us proceed to an orbifold S^1/Z_2 case. There are two orbifold fixed points, $y = 0, \pi R$. We must specify the boundary condition of the field at the fixed points in addition to the S^1 direction. By choosing the appropriate orbifolding boundary conditions, it is possible to embed the two Higgs doublets $\chi_i (i = 1, 2)$ into the zero modes $A_y^{(0)}, \Sigma^{(0)}$,

$$A_y^{(0)} = \frac{1}{2}\left(\begin{array}{cc|c} & & A_y^4 - iA_y^5 \\ & & A_y^6 - iA_y^7 \\ \hline \text{c.c.} & \text{c.c.} & \end{array}\right) \equiv \frac{1}{\sqrt{2\pi R}}\left(\begin{array}{c|c} & \frac{\chi_1}{\sqrt{2}} \\ \hline \frac{\chi_1^\dagger}{\sqrt{2}} & \end{array}\right), \tag{13}$$

$$\Sigma^{(0)} = \frac{1}{2}\left(\begin{array}{cc|c} & & \Sigma^4 - i\Sigma^5 \\ & & \Sigma^6 - i\Sigma^7 \\ \hline \text{c.c.} & \text{c.c.} & \end{array}\right) \equiv \frac{1}{\sqrt{2\pi R}}\left(\begin{array}{c|c} & \frac{\chi_2}{\sqrt{2}} \\ \hline \frac{\chi_2^\dagger}{\sqrt{2}} & \end{array}\right). \tag{14}$$

Then, the tree level potential is written, in terms of [3]

$$\Phi_1 = \frac{1}{\sqrt{2}}(\chi_1 - i\chi_2), \quad \Phi_2 = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2), \tag{15}$$

[3]The complex scalar field in the $\mathcal{N} = 2$ vector multiplet in four dimensions is given by $\phi = A_y + i\Sigma$.

as

$$
\begin{aligned}
V_{tree} &= -g^2 \mathrm{tr}[A_y^{(0)},\ \Sigma^{(0)}]^2 \\
&= \frac{g_2^2}{2}\left((\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2 - (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - \left|\Phi_1^\dagger\Phi_2\right|^2\right). \qquad (16)
\end{aligned}
$$

This is identical with the D-terms of the MSSM with $g_Y = \sqrt{3}g_4$. The relation means that the Weinberg angle is too large, $\sin^2\theta_w = 3/4$. The potential (16) has the flat direction $\Phi_1 = \Phi_2$(mod phase). If we suppose that the vacuum configuration is in the flat direction, this implies that $\tan\beta \equiv |v_2|/|v_1| = 1$, where $\Phi_{1(2)} \equiv \frac{v_{1(2)}}{\sqrt{2}}\binom{0}{1}$.

The charged and heavier neutral (CP-even) Higgses are massive at the tree level due to the quartic coupling (16),

$$
M_{H^\pm}^2 = \frac{g_2^2}{4}v^2(= M_W^2), \quad M_H^2 = \frac{g_2^2 + g_Y^2}{4}v^2 = g_2^2 v^2 (= M_Z^2). \qquad (17)
$$

In terms of the original parametrizations, we have

$$
v^2(= |v_1|^2 + |v_2|^2) = \frac{1}{g_2^2}\left((\frac{a}{R})^2 + (gp)^2\right), \qquad (18)
$$

where $\langle A_y^{(6)}\rangle = a/gR, \langle\Sigma^{(6)}\rangle = p$. Let us note that there is no g_2-dependence in the tree-level mass spectra. On the other hand, the lighter (CP-even) and CP-odd Higgses become massive at one-loop level, which are calculated by the effective potential $V_{eff}(\theta, \sigma)$, whose form is similar to (7). Their magnitude is order of $O(g_2^2)$ because they are generated at one-loop level. We also note that the corresponding SUSY breaking mass parameters $m_i^2 (i = 1, 2, 3)$ are also obtained from the effective potential.

Let us notice that one can obtain the same quartic coupling as (16) by starting with the six dimensional pure Yang-Mills theory compactified on $M^4 \times T^2/Z_2$. In this case, the two Higgs doublets are embedded in the zero modes $A_{y,z}^{(0)}$. It is easy to see that

$$
V_{tree} = -g^2 \mathrm{tr}[A_y^{(0)},\ A_z^{(0)}]^2 \qquad (19)
$$

gives the same form as (16) under the linear combinations [4],

$$
\Phi_1 = \frac{1}{\sqrt{2}}(A_y^{(0)} - iA_z^{(0)}), \quad \Phi_2 = \frac{1}{\sqrt{2}}(A_y^{(0)} + iA_z^{(0)}). \qquad (20)
$$

Both $A_y^{(0)}$ and $A_z^{(0)}$, however, correspond to the Wilson line phases, which is very different from the five dimensional case. Accordingly, the form of the effective potential is also quite different from that of the five dimensional case. There appears no Boltzmann like suppression factor in the effective potential for the present case. It is interesting to note that the nonsupersymmetric theory shares the feature of supersymmetric gauge theory through the dimensional reduction [5].

[4] The form of the quartic coupling depends on the base one chooses.

[5] It has been pointed out that the supersymmetric quantum mechanical structure is always hidden in higher dimensional gauge theories [8].

4 Conclusions

We have considered the $\mathcal{N} = 1$ vector multiplet on $M^4 \times S^1$. In order to study the vacuum structure, we have taken into account the vacuum expectation values for the adjoint scalar field, which has been overlooked in the past, in addition to the Wilson line phases and obtained the effective potential. The effect of the VEV for the adjoint scalar appears as the Boltzmann like suppression factor in the effective potential. We have found that the configuration that minimizes the effective potential does not break the gauge symmetry (10).

If we introduce hyper multiplets, we expect the vanishing VEV for Σ, but nontrivial values for θ, which is the signal for the gauge symmetry breaking. Let us note that in this case the squark field ϕ_q is also the order parameter for the gauge symmetry breaking.

In case of an orbifold S^1/Z_2, it is possible to have two Higgs doublets as the zero modes of A_y and Σ under the appropriate orbifolding boundary conditions. The tree level potential is same as the D-terms of the MSSM with $g_Y = \sqrt{3}g_4$. The lighter (CP-even) and CP-odd Higgs become massive at one-loop level. The other Higgses are massive at the tree-level due to the quartic couplings (16). The same quartic coupling is obtained from the six dimensional pure Yang-Mills theory compactified on $M^4 \times T^2/Z_2$, but in this case the two Higgs doublets correspond to the Wilson line phases, so that the form of the effective potential is different from the one for five dimensions.

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